Multi-Objective Topology Optimization of Rotating Machines Using Deep Learning

Shuhei Doi, Hidenori Sasaki, and Hajime Igarashi

Graduate School of Information Science and Technology, Hokkaido University, Sapporo 060-0814, Japan

This paper presents the fast topology optimization methods for rotating machines based on deep learning. The cross-sectional image of electric motors and their performances obtained during a multi-objective topology optimization based on the finite-element method and genetic algorithm (GA) is used for training of the convolutional neural network (CNN). Two different approaches are proposed: 1) CNN trained by preliminary optimization with a small population for GA is used for the main optimization with a large population and 2) CNN is used for screening of torque performances in the optimization with respect to the motor efficiency.

Index Terms—Deep learning (DL), genetic algorithm (GA), inner permanent magnet (IPM) motor, multi-objective optimization, topology optimization.

I. INTRODUCTION

R ECENTLY, the effective design method for electric motors, especially for electric vehicles, has been strongly required. In the optimization of electric motors, it is necessary to consider many properties such as average torque, torque ripple, iron loss, and radial force. The multi-objective optimization based on the stochastic algorithm, e.g., genetic algorithm (GA), allows obtaining Pareto solutions with respect to multiple motor properties. By increasing the population and children sizes of GA, one would obtain the sufficiently populated Pareto solutions. The optimization based on GA with a rich population, however, needs large computational cost due to a number of field computations with, e.g., the finite-element method (FEM). Thus, the solution of such multi-objective problems would lead to unallowable computational cost.

The deep learning (DL) based on convolutional neural network (CNN) has been shown effective for fast evaluation of the performance of an electric motor [1] and also the acceleration of the topology optimization [2]. In order to reduce the number of evaluations by FEM in the multi-objective optimizations, we propose here an optimization method based on DL. In this method, the cross-sectional images of electric motors and their performance obtained by a topology optimization are used for training of CNN. Then, CNN is used as the surrogate method of FEM for the acceleration of the multi-objective topology optimization. Two different approaches are proposed here: 1) CNN trained by a preliminary optimization with a small population of GA is used for the main optimization with a large population and 2) CNN trained by an optimization with respect to torque performances is used for screening of torque performances in another optimization with respect to the motor efficiency. The first approach is an effective way to obtain a sufficiently populated Pareto solution using GA with a rich population. The second approach can be generalized as follows: CNN is trained through data obtained by solving problem A, and then, the trained CNN is extendedly used for fast optimization of problems B, C,... with different cost functions or constraints. These approaches will be verified by numerical examples.

II. MULTI-OBJECTIVE TOPOLOGY OPTIMIZATION WITH CNN

A. Why Deep Learning

In this paper, the data composed of the cross-sectional images of electric motors and their performance obtained during the topology optimization process are used for training of CNN, which is then used to reduce the field computing cost of optimization with other cost functions, constraints, and models. There exist several approximate computing methods for optimization that are used for evaluation of fitness instead of time-consuming field computations using, e.g., FEM. They include the response surfaces [3], [4], kriging method [5], and artificial neural network (ANN) [6]. Although the former two methods are effective when the degree of freedoms (DoFs) are less than, say, ten, DoFs in the pixel image of an electric motor are far larger. ANN can treat the images, while the image features for accurate classification have to be extracted by the user. However, it would be hard to extract such features from diverse images generated through the topology optimization. In contrast, CNN can automatically extract such features from the training images. For these reasons, we employ CNN for the surrogate model in this paper.

B. Classifier Using CNN

In the learning phase, we construct the classifiers of the average torque $T_{ave}$ and torque ripple $T_{rip} = (T_{max} - T_{min})/T_{ave}$ using CNN with 22 layers, for which GoogLenet is employed [7]. The structure of CNN is schematically shown in Fig. 1. The input data for CNN are a bitmap image representing the cross-sectional shape of an inner permanent magnet (IPM) motor. The output data are the corresponding
The classes provided by CNN are summarized in Tables I and II, where $T_{\text{ave}}^{\text{FEM}}$ and $T_{\text{rip}}^{\text{FEM}}$ denote the average torque and torque ripple computed with FEM, while $T_{\text{ave}}^{\text{CNN}}$ and $T_{\text{rip}}^{\text{CNN}}$ denote the classes output from CNN. The training data are obtained by a preliminary multi-objective topology optimization of the IPM motor shown in Fig. 2 with relatively small populations in GA. In the topology optimization, element state $S_e$ is determined from the shape function $\phi(x) = \sum_{i=1}^{n} w_i G_i(x)$, where $G_i$ and $w_i$ denote the normalized Gaussian function and weighting coefficient such that $S_e = \text{iron}(\text{air}) \text{if } \phi \geq 0(<0)$ [8]. The optimization problem is defined as follows:

$$\max T_{\text{ave}}(w)$$

$$\min T_{\text{rip}}(w)$$

where $w$ denotes the vector composed of $w_i, i = 1, 2, \ldots, n$. Note that, the material distribution and thus machine performance depend on $w$. The population sizes are summarized in the left column of Table III. Fig. 3 shows the distribution of $T_{\text{ave}}^{\text{CNN}}$ in the initial population, whose individuals are randomly generated. As shown in Fig. 3, many individuals belong to the classes with low average torques. Such individuals could be approximately evaluated only by CNN because these have little influence on the evolution. On the other hand, individuals in classes with higher torques should be accurately evaluated by FEM. Namely, the trained CNN can be effectively used for screening of the torque performances.

We use 6000 data obtained during the multi-objective optimization of the IPM motor with respect to $T_{\text{ave}}$ and $T_{\text{rip}}$ using GA with the size parameters in the left column of Table III for the training of CNN. The 700 data that are not used for the training are used for the test of CNN. Tables IV and V summarize the classification results. It is found that there are a small number of off-diagonal samples. The reason why the classification accuracy in $T_{\text{rip}}$ is lower than that in $T_{\text{ave}}$ is due to the relatively weak correlation between the material distribution and torque ripple.

C. Optimization Method (i)

We use the CNN trained for the data obtained through the preliminary optimization process mentioned above for the acceleration of the main multi-objective topology optimization with larger population size, which is summarized in the center column of Table III. The optimization model is shown in Fig. 4.

In the main multi-objective optimization process after the learning phase, the values of $T_{\text{ave}}^{\text{CNN}}$ and $T_{\text{rip}}^{\text{CNN}}$ are evaluated by the trained CNN to make fast evaluation of the rank $F_j^{\text{CNN}} \in Z$ of individual $i$ using non-dominated sorting by NSGA-II [9]. Since there would be errors in the ranking, the correction is made by performing the finite element (FE) analysis with

<table>
<thead>
<tr>
<th>Table III</th>
<th>PARAMETERS FOR GA</th>
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<tbody>
<tr>
<td>Preliminary</td>
<td>Problem (i)</td>
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<tr>
<td>Number of genes</td>
<td>28</td>
</tr>
<tr>
<td>Population size $N_{\text{pop}}$</td>
<td>192</td>
</tr>
<tr>
<td>Number of children $N_c$</td>
<td>64</td>
</tr>
<tr>
<td>Number of parents $N_p$</td>
<td>30</td>
</tr>
<tr>
<td>Number of generations</td>
<td>100</td>
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</tbody>
</table>
TABLE IV  
CLASSIFIER OF AVERAGE TORQUE

<table>
<thead>
<tr>
<th>Accuracy (%)</th>
<th>CNN</th>
<th>FEM</th>
</tr>
</thead>
<tbody>
<tr>
<td>92%</td>
<td>0.25 0.75 1.25 1.6 2 4.2</td>
<td>0 0 0 0 0 0</td>
</tr>
<tr>
<td>78%</td>
<td>0.25 0.8 1.25 2 4.2</td>
<td>0 0 0 0 0 0</td>
</tr>
<tr>
<td>60%</td>
<td>1.25 0.8 1.25 5 0 0 0</td>
<td>0 0 0 0 0 0</td>
</tr>
<tr>
<td>50%</td>
<td>1.25 0.8 1.25 5 0 0 0</td>
<td>0 0 0 0 0 0</td>
</tr>
<tr>
<td>40%</td>
<td>1.25 0.8 1.25 5 0 0 0</td>
<td>0 0 0 0 0 0</td>
</tr>
</tbody>
</table>

TABLE V  
CLASSIFIER OF TORQUE RIPPLE

<table>
<thead>
<tr>
<th>Accuracy (%)</th>
<th>CNN</th>
<th>FEM</th>
</tr>
</thead>
<tbody>
<tr>
<td>92%</td>
<td>0.11 0.3 0.6 0.9 1.5</td>
<td>0 0 0 0 0</td>
</tr>
<tr>
<td>80%</td>
<td>0.11 0.3 0.6 0.9 1.5</td>
<td>0 0 0 0 0</td>
</tr>
<tr>
<td>70%</td>
<td>0.11 0.3 0.6 0.9 1.5</td>
<td>0 0 0 0 0</td>
</tr>
<tr>
<td>60%</td>
<td>0.11 0.3 0.6 0.9 1.5</td>
<td>0 0 0 0 0</td>
</tr>
<tr>
<td>50%</td>
<td>0.11 0.3 0.6 0.9 1.5</td>
<td>0 0 0 0 0</td>
</tr>
</tbody>
</table>

The probability $P_i$ given by

$$P_i = \left( \frac{F_{\text{CNN worst}} - F_{\text{CNN i}}}{F_{\text{CNN worst}} - 1} \right)^2$$

where $P_i$ is evaluated over the whole population in the first generation and over the children in other generations. We design $P_i$ so that it increases as the score provided by CNN becomes better. In Fig. 5, $P_i$ is plotted against $F_{\text{CNN i}}$.

In the first step of GA, a limited number of individuals would be analyzed by FEM because the majority of the population tends to have low values of $F_{\text{CNN i}}$. The algorithm of the main multi-objective optimization is shown in Fig. 6.

D. Optimization Method (ii)

We consider here the optimization to reduce the iron loss of the IPM motor keeping the torque performance better than a threshold. To make the computing cost smaller, CNN trained for the preliminary optimization is used for screening of the torque performances. The optimization problem is defined by

$$\min W_{\text{iron}},$$

subjected to $T_{\text{ave}} > 2.0$ and $T_{\text{rip}} < 0.2$  

where $W_{\text{iron}}$ denotes the iron loss, which is evaluated here by the Steinmetz formula

$$W_{\text{iron}} = k_h f \left( \frac{B_a}{B_0} \right)^2 + k_e f^2 \left( \frac{B_a}{B_0} \right)^2$$

where $B_a$ and $f$ denote the maximum amplitude and frequency of the magnetic flux density, and the constants are set as follows: $B_0 = 1$, $k_h = 7.5 \times 10^{-2}$ J, and $k_e = 6.0 \times 10^{-4}$ J · s.

In this optimization, we evaluate $T_{\text{ave}}$ and $T_{\text{rip}}$ using the trained CNN, and if they satisfy the prescribed condition

$$T_{\text{ave}} \geq 1.7 \text{ and } T_{\text{rip}} \leq 0.4,$$

these torque properties as well as iron loss are evaluated by FEM. The region satisfying this condition is shown in Fig. 7. We impose this weakened condition for the FE analysis because the original constraint in (3) is too strict to have a sufficiently large number of population that evolves toward the optimal solution. In the first step of the GA process, the majority of the population does not satisfy (5). Thus, the number of the FE analysis is strongly suppressed. The GA setting is summarized in the right column of Table III.

III. OPTIMIZATION RESULTS

A. Result Obtained by Method (i)

Fig. 8 shows the Pareto solutions at the 100th generation of GA obtained by the preliminary and main multi-objective optimizations. Using the proposed method, the Pareto solutions can be refined with reduced computational cost.
Fig. 7. Domain for the execution of FE analysis.

Fig. 8. Comparison of Pareto solutions obtained by method (i).

Fig. 9. Rate of FEM evaluations.

The number of the FE analysis, normalized by the total number of ranking evaluation, is plotted against the GA generations shown in Fig. 9. Because the individuals with higher ranking increase with the generation, the number of FE evaluations increases. In the entire optimization process, the FE analysis is performed for 49.5% of the individuals.

The IPM motors corresponding to the solutions on the Pareto front, marked by A and B in Fig. 8, are shown in Figs. 10 and 11, where solution A has the largest average torque and largest torque ripple, while B has a good balance in these two torque performances. The rotor of solution A has large notches which would yield large reluctance torque as well as large torque ripple. On the other hand, the rotor of solution B has a smooth surface and hence has a lower torque ripple and average torque.

B. Result Obtained by Method (ii)

Fig. 12 shows the optimization result, which is found to meet the constraints. The flux distribution in the air gap is shown in Fig. 13. The iron loss, average torque, and torque ripple of the best individual are plotted against the generation of GA shown in Fig. 14(a)–(c). The iron loss is not plotted until 21st generation because no individuals are analyzed by FEM. The torque ripple goes up and down because the optimization seeks for a ground for a compromise between the average torque and torque ripple which have a reciprocal relationship.

The number of FE analysis, normalized by the total number of ranking evaluation, is plotted against the generation of
In method (i), the CNN trained through the preliminary optimization with a small population is used to make a fast evaluation of fitness in the main optimization with a large population. In method (ii), the CNN trained through the preliminary optimization with respect to the torque performances is used for fast evaluation of constraints in the optimization with respect to the iron loss. The FE computations are reduced to about 50% and 30%, respectively, by methods (i) and (ii).

We plan to study methods for adequate design of the probability function and the condition for the FE analysis. We will also study the adequate modification of the shape obtained by the topology optimization for industrial realization.

REFERENCES